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A Dominant-Data Matching Method for Digital Control Systems Modeling and Design

L. S. SHIEH, Y. F. CHANG, AND R. E. YATES

Abstract—**A dominant-data matching method is presented for obtaining a reduced-order pulse-transfer function from either a high-order continuous-data transfer function or a high-order pulse-transfer function, and for identifying the pulse-transfer function of a system from available experimental time and frequency response data. The method may also be applied to the digital control systems design problem with various sampling periods.**

I. INTRODUCTION

Most practical industrial circuits and control systems are continuous-time systems for which analog filters and controllers are employed to improve performance. The recent availability of high-performance low-cost microprocessors and associated electronics has led to replacement of many continuous systems with system employing digital filters and controllers. Many techniques have been developed for digital control systems design [1]–[4]. Among them the ω-domain bilinear transformation is often applied to design industrial digital controllers. However, this method is graphical and involves "cut-and-try" procedures. Recently, Kuo [5] and others developed an optimal discrete-time data matching method for the redesign of a continuous-data system. Constant controllers instead of dynamic digital controllers are mainly employed in these designs. As a result, good performances of redesigned systems can be achieved if the frequency of the input signal is sufficiently lower than the sampling frequency. As an alternate to Kuo's time-domain approach, Rattan and Yeh [6] have given an elegant frequency-domain method for the redesign of continuous-data systems. The method of weighted least squares complex-curve fitting due to Levy [7] and Sanathanan and Koerner [8] has been successfully extended in the z-domain to determine a dynamic digital controller. As a result of these efforts, better performance of redesigned systems can be achieved. In this correspondence, a computer-aided method is proposed for matching the dominant data of a high-order continuous-data system, or a discrete-data system, with the dominant response of a low-order digital system replacement. Also, methods are given for system identification and digital controller design of these systems.

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II. DOMINANT-DATA MATCHING METHOD

The characteristics of a control system or a filter are often expressed by either a time-response curve, or a frequency-response curve or a set of poles and zeros in the complex plane, or both. The quantitative description of transient behavior is represented by its time-domain control specification [9] (for example, the percentage overshoot and the rise time) and by the frequency-domain control specifications (for example, the maximum value of the closed-loop frequency response and the bandwidth). These specifications which are defined for analog control systems [9] can be considered as specifications of digital control systems. This is because a digital control system can be viewed as a continuous-data system in the frequency domain when $z = e^{j\omega T}$ where T is a sampling period.

Some empirical observations or rules of thumb due to Axelby [10] and Truxal [11] that link the specifications of the continuous-data systems in both the time and frequency domains can be found in [10], [11].

Recently Shieh *et al.* [12] have studied the relationships between the complex-domain specifications (the damping ratio and the undamped natural angular frequency) and the time-domain and frequency-domain specifications. Based on the results [12], the time and complex-domain control specifications can be converted into the equivalent frequency-domain specifications and vice versa. These control specifications are used as dominant data of a continuous or discrete-data control system.

Our new dominant-data matching method matches the dominant data of a continuous-data or a discrete-data system to those of the newly designed or modeled discrete-data system. The steps involved are as follows:

Step 1: Determine a set of dominant frequency-response data from the assigned or obtained time-domain and complex-domain specifications by using the rules and results in [10]–[12].

Step 2: Assume a fixed configuration digital system and controllers with unknown constants. Determine the open-loop and the overall pulse-transfer function of the system.

Step 3: Formulate a set of linear/nonlinear equations by matching the unknown constants of the pulse-transfer function and the assigned dominant data. Solve the equations by using the multidimensional Newton–Raphson method [13], available as a library computer program package (called the Z system) in many digital computers [14].

Step 4: Estimate initial value for the numerical solution of the Newton–Raphson method by constructing a crude pulse-transfer function which can be obtained by a complex-curve fitting method.

Step 5: Compare the results with the assigned specifications.

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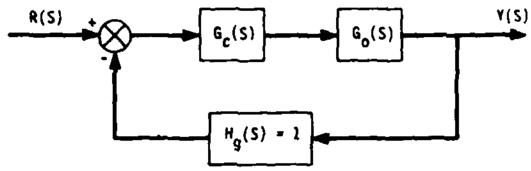

Fig. 1. Block diagram of a missile pitch control system.

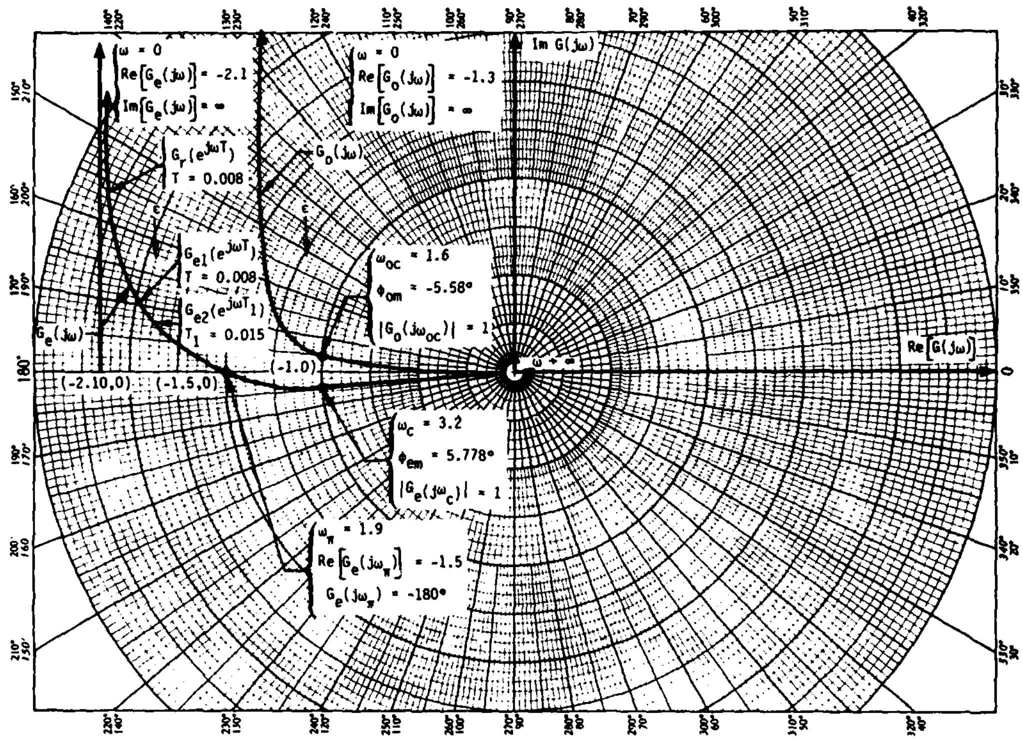

Fig. 2. The Nyquist plots of various example open-loop transfer functions.

III. MODELING A REDUCED-ORDER PULSE-TRANSFER FUNCTION

We use a real stabilized pitch control system of a semiactive terminal homing missile [15] as an illustrative model to show that the characteristics of the transient-state response of a system can be estimated from the dominant frequency-response data and the applications of the proposed method to the identification and model reduction problems. A block diagram of the missile system is shown in Fig. 1. The closed-loop high-order transfer function is

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G_0(s)}{1 + G_c(s)G_0(s)H_g(s)} = \frac{G_e(s)}{1 + G_e(s)} \triangleq T_e(s) \tag{1a}$$

where

$G_c(s)$ = stabilization filter

$$= \frac{1.6\left(\frac{s}{25}+1\right)\left(\frac{s}{125}+1\right)}{\left[\left(\frac{s}{150}\right)^2 + \left(\frac{0.6}{150}\right)s + 1\right]\left[\left(\frac{s}{200}\right)^2 + \left(\frac{0.8}{200}\right)s+1\right]} \tag{1b}$$

$G_0(s)$ = transfer function of the actuator and aerodynamics of the missile system

$$= \frac{324\,332.316(s + 0.1933)(s + 65)(s + 1500)}{s(s-2.921)(s+3.175)(s+87.9 \pm j95.5)(s+112.5)(s+1385)} \tag{1c}$$

$$H_g(s) = \text{transfer function of the gyro} = 1 \tag{1d}$$

$$G_e(s) \triangleq G_c(s)G_0(s) = \text{unstable open-loop transfer function of the existing stabilized system.} \tag{1e}$$

The Nyquist plots of $G_e(s)$ and $G_0(s)$ are shown in Fig. 2. The dominant data of $G_e(s)$ are:

1) Real and imaginary parts of $G_e(s)$ at $s = j\omega = j0$ are

$$\text{Re}\,[G_e(j0)] = -2.103\,817$$
$$\text{Im}\,[G_e(j0)] = \infty. \tag{2a}$$

2) Gain margin G_{em} of this system $G_e(j\omega_\pi)$ is

$$G_{em} = \left|\frac{1}{G_e(j\omega_\pi)}\right| = \left|\frac{1}{\mathrm{Re}\,[G_e(j\omega_\pi)]}\right| \cong \left|\frac{1}{-1.5}\right| \tag{2b}$$

where the phase-cross-over frequency ω_π is given by

$$\omega_\pi \cong 1.9 \text{ rad/s, such that } \angle G_e(j\omega_\pi) = -180^\circ. \tag{2c}$$

The equivalent real and imaginary parts of $G_e(j\omega_\pi)$ at $\omega_\pi = 1.9$ rad/s are

$$\mathrm{Re}\,[G_e(j\omega_\pi)] = -1.507\,944 \tag{2d}$$

$$\mathrm{Im}\,[G_e(j\omega_\pi)] = -0.006\,490\,205. \tag{2e}$$

3) Phase margin ϕ_{em} of the system $G_e(j\omega)$ is

$$\phi_{em} = 180^\circ + \angle G_e(j\omega_c) \cong 5.7787^\circ \tag{2f}$$

where the gain-cross-over frequency ω_c is given by $\omega_c \cong 3.2$ rad/s so that

$$|G_e(j\omega_c)| = 1. \tag{2g}$$

Equivalent real and imaginary parts of $G_e(j\omega_c)$ are

$$\mathrm{Re}\,[G_e(j\omega_c)] = -0.993\,914\,3 \tag{2h}$$

$$\mathrm{Im}\,[G_e(j\omega_c)] = -0.095\,474\,78. \tag{2i}$$

It is required to determine a reduced-order pulse-transfer function such that the characteristics of the identified discrete-data model agree as closely as possible with those of the high-order continuous-data system.

Let the required overall pulse-transfer function be

$$T_r(z) = \frac{G_r(z)}{1 + G_r(z)} \tag{3a}$$

where the open-loop pulse-transfer function $G_r(z)$ is

$$G_r(z) = \frac{x_0 z^p + x_1 z^{p-1} + \cdots + x_{p-1} z + x_p}{(z-1)(y_0 z^q + y_1 z^{q-1} + \cdots + y_{q-1} z + y_q)}. \tag{3b}$$

$G_r(z)$ is assigned to be a type "1" system because $G_e(s)$ in (1e) is a type "1" system. To match the five dominant data in (2) we choose $q = 2$, $p = 2$, and $y_0 = 1$. Thus $G_r(z)$ becomes

$$G_r(z) = \frac{x_0 z^2 + x_1 z + x_2}{(z-1)(z^2 + y_1 z + y_2)} \tag{4}$$

where x_i and y_i in (4) are unknown constants to be determined. The goal is to determine the unknown constants x_i and y_i in $G_r(z)$ so that $G_r(z)$ as $z = e^{j\omega T}$ matches the dominant data in (2). The sampling period T (=0.008 s) and ω_s (=250π rad/s) are chosen to be synchronized with the 125-Hz pulse-width modulated actuator [16]. Since $G_e(s)$ and $G_r(z)$ are type "1" systems and we need to match the dominant data of $G_e(j\omega)$ as $\omega = 0$ in (2a) and the dominant data of $G_r(z)$ as $\omega = 0$ or $z = e^{j\omega T} \simeq 1$ in (3b) or (4), the expression for $G_r(z)$ in (3b) is modified as follows:

Substituting $z = z^* + 1$ into (3b) yields

$$G_r(z^*) = \frac{x_p^* + x_{p-1}^* z^* + \cdots + x_0^* z^{*p}}{z^*(y_q^* + y_{q-1}^* z^* + \cdots + y_0^* z^{*q})}$$
$$= e_{-1} z^{*-1} + e_0 + e_1 z^* + e_2 z^{*2} + \cdots \tag{5a}$$

where

$$x_p^* = \sum_{i=0}^{p} x_i, \quad x_{p-1}^* = \sum_{i=1}^{p} i x_{p-i}, \quad y_q^* = \sum_{i=0}^{q} y_i,$$

$$y_{q-1}^* = \sum_{i=1}^{q} i y_{q-i}$$

$$e_{-1} = x_p^*/y_q^* \quad \text{and } e_0 = (y_q^* x_{p-1}^* - y_{q-1}^* x_p^*)/y_q^{*2}, \text{ etc.}$$

Equating the respective real and imaginary parts of $G_r(z)$ for $\omega = 0$ and those of $G_r(z^*)$ for $\omega = 0$ gives

$$\mathrm{Re}\,[G_r(z)]\,|_{z=1+j0} = \mathrm{Re}\,[G_r(z^*)]\,|_{z^*=z-1=j0} = e_0 \tag{5b}$$

and

$$\mathrm{Im}\,[G_r(z)]\,|_{z=1+j0} = \mathrm{Im}\,[G_r(z^*)]\,|_{z^*=z-1=j0} = \infty. \tag{5c}$$

Equations (5b) and (5c) imply that e_0 in (5b) is the asymptotic line of the type "1" systems at low frequencies.

In the frequency domain, (4) can be expressed in an alternative form as follows:

Let us define

$$z = e^{j\omega_k T} = \cos\omega_k T + j\sin\omega_k T \triangleq u_k + jv_k \tag{6a}$$

and substituting $z = u_k + jv_k$ into (4), we have

$$G_r(u_k, v_k) = \frac{(x_0 u_k^2 - x_0 v_k^2 + x_1 u_k + x_2) + j(2x_0 u_k v_k + x_1 v_k)}{[(u_k - 1)(u_k^2 - v_k + y_1 u_k + y_2) - v_k(2u_k v_k + y_1 v_k)] + j[(u_k - 1)(2u_k v_k + y_1 v_k) + v_k(u_k^2 - v_k^2 + y_1 u_k + y_2)]} = R_k + jI_k \tag{6b}$$

where ω_k are specific frequencies and $R_k \triangleq \mathrm{Re}\,[G_r(u_k, v_k)]$, $I_k \triangleq \mathrm{Im}\,[G_r(u_k, v_k)]$. If R_k and I_k are the known or assigned values at frequencies ω_k, we can obtain two linear equations. First, we multiply both sides of (6b) by the common denominator, then we separate the real and imaginary parts and then equate the respective real and imaginary parts. Thus we have

$$\begin{aligned} f_r(x_0, x_1, x_2, y_1, y_2) &= (x_0 u_k^2 - x_0 v_k^2 + x_1 u_k + x_2) \\ &\quad - R_k[(u_k - 1)(u_k^2 - v_k^2 + y_1 u_k + y_2) \\ &\quad - v_k(2u_k v_k + y_1 v_k)] + I_k[(u_k - 1)(2u_k v_k \\ &\quad + y_1 v_k) + v_k(u_k^2 - v_k^2 + y_1 u_k + y_2)] \\ &= 0 \end{aligned} \tag{6c}$$

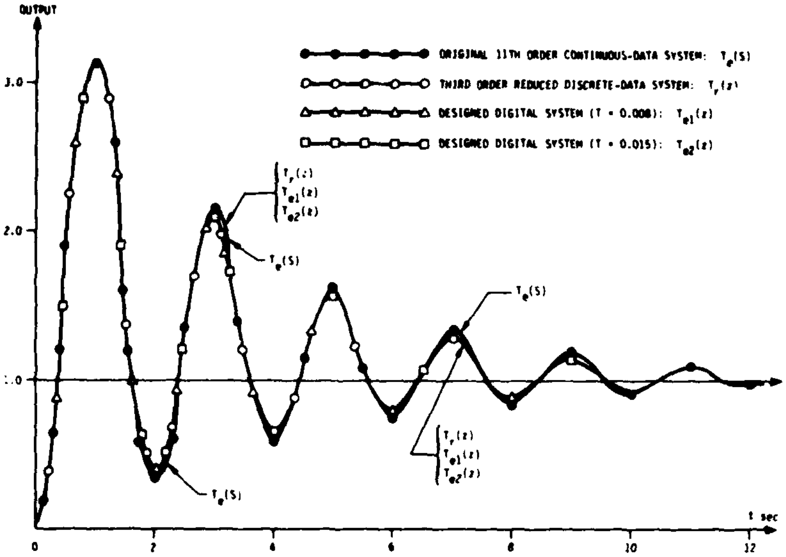

Fig. 3. *Time responses of the various systems.*

and

$$
\begin{aligned}
&f_{i+1}(x_0, x_1, x_2, y_1, y_2)\\
&\quad= (2x_0 u_k v_k + x_1 v_k)\\
&\qquad - R_k[(u_k - 1)(2u_k v_k + y_1 v_k) + v_k(u_k^2 - v_k^2\\
&\qquad + y_1 u_k + y_2)] - I_k[(u_k - 1)(u_k^2 - v_k^2\\
&\qquad + y_1 u_k + y_2) - v_k(2u_k v_k + y_1 v_k)]\\
&\quad= 0.
\end{aligned}
\tag{6d}
$$

Using the expressic in (5b), (6c), and (6d) and the assigned dominant data in \, (2d), (2e), (2h), and (2i), we can formulate one nonli equation and four linear equations $f_i(x_i, y_i) = 0$ for $i = 1, \cdots, 5$.

The Newton–Raphson method available as a library computer program in most digital computers [14] can be applied to solve these nonlinear equations. However, as is well known, the Newton–Raphson method will converge to a desired solution for a small range of starting values or initial solution estimates. To improve the convergence and to obtain the set of desired solutions, we offer the following method for initial estimates.

Since $f_1(x_i, y_i) = 0$ is nonlinear and $f_i(x_i, y_i) = 0$, $i = 2, \cdots, 5$ are linear equations, we linearize $f_1(x_i, y_i) = 0$ by choosing a very low frequency. For example, if we choose $\omega_k = 0.01 \triangleq \omega_{0.01}$, then $R_k \triangleq R_{0.01} = -2.1$, $I_k \triangleq I_{0.01} = 40.173\,19$, $u_k \triangleq u_{0.01} = \cos \omega_{0.01} T = 0.999\,999\,50$, and $v_k \triangleq v_{0.01} = \sin \omega_{0.01} T = 8 \times 10^{-5}$. Solving $f_1(x_i, y_i) = 0$ and $f_i(x_i, y_i) = 0$, $i = 2, \cdots, 5$ for the unknown constants x_i (defined as x_i^*) and y_i (defined as y_i^*) we have the initial estimates x_i^* and y_i^*. Using these values as initial estimates for the solutions of $f_i(x_i, y_i) = 0$ and using the Newton–Raphson method we obtain the exact solution x_i and y_i at the second iteration with error tolerance of 10^{-6}. The desired open-loop pulse-transfer function is

$$
G_r(z) = \frac{0.006\,792\,596z^2 - 0.012\,335\,992z + 0.005\,545\,311\,4}{z^3 - 2.998\,541\,2z^2 + 2.996\,486\,4z - 0.007\,945\,2}.
\tag{7}
$$

A Nyquist plot of $G_r(z)$ is shown in Fig. 2. The plot matches closely that of $G_e(s)$ not only at the dominant frequencies, but also at others. The $G_r(z)$ is seen to be a good reduced-order digital model of the original 11th-order unstable analog system $G_e(s)$. This is the contribution of our new method because there are no known effective continuous-discrete model conversion and model reduction methods for unstable systems. The resulting closed-loop pulse-transfer function which is the reduced-order discrete-data model of the original high-order continuous data system is

$$
\begin{aligned}
T_r(z) &= \frac{G_r(z)}{1 + G_r(z)}\\
&= \frac{0.006\,792\,596z^2 - 0.012\,335\,992z + 0.005\,545\,311\,4}{z^3 - 2.991\,748\,524z^2 + 2.984\,150\,408z - 0.992\,399\,888\,6}.
\end{aligned}
\tag{8}
$$

Since the assigned dominant data are the steady-state frequency response, it is interesting to compare responses of the 11th-order continuous function $T_e(s)$ in (1a) and the 3rd-order discrete function $T_r(z)$ in (8) shown in Fig. 3. Observe that both the transient response and steady-state response of the reduced-order model $T_r(z)$ are excellent matches of the original high-order system. This indicates that the dynamic characteristics of the system (for example, peak value time and overshoot, which may not occur at the sampling time) are indirectly controlled by the assignment of the gain-cross-over frequency and the phase margin. This is a major advantage of our new method. Also note that the reduced-order model gives an excellent approximation of the original system when driven by high-frequency input signals.

To determine the initial estimates x_i^* and y_i^*, a general formulation of a set of linear equations can be constructed from the following complex-curve fitting method.

Consider the pulse-transfer function

$$
G(z) = \frac{x_0^* z^m + x_1^* z^{m-1} + \cdots + x_m^*}{y_0^* z^n + y_1^* z^{n-1} + \cdots + y_n^*}
\tag{9a}
$$

where $y_0^* = 1$ and x_l^* and y_l^* are unknown constants to be determined. Substituting $z^r = e^{jr\omega_k T} = \cos r\omega_k T + j \sin r\omega_k T$ into (9a) gives

$$G(e^{jr\omega_k T}) = \frac{\sum_{l=0}^{m} x_l^* \cos (m-l)\omega_k T + j \sum_{l=0}^{m} x_l^* \sin (m-l)\omega_k T}{\sum_{l=0}^{n} y_l^* \cos (n-l)\omega_k T + j \sum_{l=0}^{n} y_l^* \sin (n-l)\omega_k T}$$

$$= R(\omega_k) + jI(\omega_k) = R_k + jI_k \tag{9b}$$

where R_k and I_k are the real and imaginary parts of the transfer function at the experimental frequencies or the assigned frequencies ω_k. After multiplying both sides of (9b) by the common denominator and separating the real and imaginary parts, we equate the respective real and imaginary parts. This yields the following matrix equation:

$$\begin{bmatrix} \cos m\omega_1 T \cos (m-1)\omega_1 T & \cdots & 1 & (-R_1 \cos (n-1)\omega_1 T + I_1 \sin (n-1)\omega_1 T) & \cdots & (-R_1 \cos \omega_1 T + I_1 \sin \omega_1 T) & -R_1 \\ \sin m\omega_1 T \sin (m-1)\omega_1 T & \cdots & 0 & (-R_1 \sin (n-1)\omega_1 T - I_1 \cos (n-1)\omega_1 T) & \cdots & (-R_1 \sin \omega_1 T - I_1 \cos \omega_1 T) & -I_1 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ \cos m\omega_i T \cos (m-1)\omega_i T & \cdots & 1 & (-R_i \cos (n-1)\omega_i T + I_i \sin (n-1)\omega_i T) & \cdots & (-R_i \cos \omega_i T + I_i \sin \omega_i T) & -R_i \\ \sin m\omega_i T \sin (m-1)\omega_i T & \cdots & 0 & (-R_i \sin (n-1)\omega_i T - I_i \cos (n-1)\omega_i T) & \cdots & (-R_i \sin \omega_i T - I_i \cos \omega_i T) & -I_i \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_0^* \\ x_1^* \\ \cdot \\ x_m^* \\ y_1^* \\ y_2^* \\ \cdot \\ y_n^* \end{bmatrix} = \begin{bmatrix} (R_0 \cos n\omega_0 T - I_0 \sin n\omega_0 T) \\ (R_0 \sin n\omega_0 T + I_0 \cos n\omega_0 T) \\ \vdots \\ (R_i \cos n\omega_i T - I_i \sin n\omega_i T) \\ (R_i \sin n\omega_i T + I_i \cos n\omega_i T) \\ \vdots \end{bmatrix}. \tag{9c}$$

Substituting the selected $(n + m + 1)$ frequency-response data into (9c), we can solve for the required $(n + m + 1)$ unknown constants x_l^* and y_l^*.

IV. DIGITAL CONTROL SYSTEM DESIGN

Consider the pitch control transfer function of the missile system of (1a). The unity-feedback system without the stabilization filter $G_c(s)$ is unstable, and a rate gyro is not available for this example system. It is required to design a digital controller $G_c(z)$ instead of an analog controller $G_c(s)$ such that the designed system has the exact control specifications [9] of the original stabilized continuous-data system given in (2). Furthermore, the response $G_e(j\omega)$ at $\omega = 140$ rad/s $\triangleq \omega_{140}$ is chosen as a dominant data constraint because the system has an inherent high frequency signal component at ω_{140}. This is a digital redesign problem. The structure of the digital control system is shown in Fig. 4. The closed-loop pulse-transfer function of the desired digital system becomes

$$\frac{Y(z)}{R(z)} = \frac{G_c(z)G_hG_0(z)}{1 + G_c(z)G_hG_0(z)} \triangleq T_{e1}(z) = \frac{G_{e1}(z)}{1 + G_{e1}(z)} \tag{10}$$

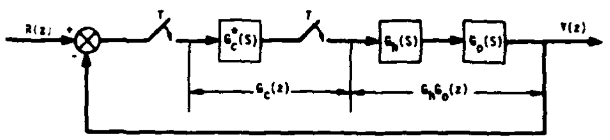

Fig. 4. Block diagram of a digital pitch control system.

where

$$G_hG_0(z) = (1 - z^{-1})Z\left[\frac{1}{s} G_0(s)\right]$$

$$G_{e1}(z) \triangleq G_c(z)G_hG_0(z)$$

$$G_h(s) = \frac{1 - e^{-sT}}{s} = \text{the zeroth-order hold}$$

T = the sampling period = 0.008 s.

$G_c(z)$ is the desired digital controller and is

$$G_c(z) = \frac{x_0 z^2 + x_1 z + x_2}{z^2 + y_1 z + y_2} \tag{11}$$

where x_l and y_l are unknown constants to be determined. Because $G_c(z)$ is a forward controller, the equations $f_i(x_l, y_l) = 0$ can be formulated from the following:

$$G_c(z)|_{z=e^{j\omega_k T}} = \frac{G_e(s)|_{s=j\omega_k}}{G_hG_0(z)|_{z=e^{j\omega_k T}}} \tag{12}$$

where $\omega_0 = 0$, $\omega_{1.9} = 1.9$, $\omega_{3.2} = 3.2$, and $\omega_{140} = 140$. Notice that $G_hG_0(z)$ for $z = e^{j\omega_k T}$ is not equal to $G_0(s)$ for $s = j\omega_k$ unless $T = 0$. Using the dominant data of (2a), (2h), and (2i), the required response at ω_{140}, or Re $[G_c(e^{j\omega_{140} T})]$ = 26.951 878 and Im $[G_c(e^{j\omega_{140} T})]$ = 19.196 865, and the relationships expressed in (5b) and (12) yield a set of equations $f_i(x_l, y_l) = 0$ for $i = 1, 2, \cdots, 5$.

The set of linear and nonlinear equations can be solved using the Newton–Raphson method. The initial estimates

for the Newton–Raphson solution may be determined from (9c). Another linear equation $f_1^*(x_l, y_l) = 0$, instead of $f_1(x_l, y_l) = 0$ can be constructed to yield five linear equations with five unknown constants (x_l^* and y_l^*). $G_e(j\omega)$ at $\omega = 0.01 \triangleq \omega_{0.01}$ is used in this case. Substituting Re $[G_c(e^{j\omega_{0.01}T})] = 1.596\,112\,0$ and Im $[G_c(e^{j\omega_{0.01}T})] = 6.409\,642$ into (12) we get the linear equation. Using x_l^* and y_l^* obtained from (9c) and the Newton–Raphson method, we obtain the solution x_l and y_l of $f_i(x_l, y_l) = 0$ at the fifth iteration with the error tolerance of 10^{-6}. The required digital compensator is

$$G_c(z) = \frac{11.869\,083z^2 - 13.492\,37z + 3.058\,4008}{z^2 - 0.750\,552\,99z + 0.646\,992\,37} = \frac{11.869\,083(z - 0.824\,080\,67)(z - 0.312\,685\,33)}{(z - 0.375\,276\,50 + j0.711\,449\,17)(z - 0.375\,276\,50 - j0.711\,449\,17)}. \tag{13}$$

The Nyquist plot of $G_{e1}(z) \triangleq G_c(z)G_hG_0(z)$ is shown in Fig. 2. It closely matches the Nyquist plot of $G_e(s)$. The unit-step responses of the existing stabilized continuous-data system $T_e(s)$ in (1a) and $T_{e1}(z)$ in (10) are shown for comparison in Fig. 3. The time-response of the newly designed sampled-data system is very close to the existing stabilized system. It is interesting to note that $G_c(s)$ of (1b) is a fourth-order analog controller whereas the $G_c(z)$ of (13) is a second-order digital controller.

In a large control system it is often difficult to select a minimum common sampling period among the various subsystems. For example, the missile inner loop stability system with sampling period $T = 0.008$ s is used with a terminal guidance system. The terminal guidance system is low pass. Therefore, a larger sampling period may be economically used in this system. If we assign a larger sampling period T_1 (=0.015 s) for the outer guidance loop, and we desire a single sample period, we must raise the sampling period T (=0.008 s) of the actuator and inner loop from T (=0.008) to T_1 (=0.015). Notice that the new sampling frequency ω_{s1} $(=2\pi/T_1 = 418.88) \gg 2\omega_{140}$ (=280 rad/s). The modified open-loop pulse transfer function with $T_1 = 0.015$ s is

$$G_hG_0^*(z) \triangleq Z\left[\frac{1 - e^{-sT_1}}{s} G_0(s)\right]. \tag{14}$$

Since a larger sampling period T_1 is used, we select a third-order digital controller $G_c^*(z)$ rather than the second-order digital controller. $G_c(z)$ of (13) is

$$G_c^*(z) = \frac{x_0z^3 + x_1z^2 + x_2z + x_3}{z^3 + y_1z^2 + y_2z + y_3}. \tag{15}$$

The x_l and y_l are seven unknown constants to be determined. $G_e(j\omega)$ at $\omega = 0 \triangleq \omega_0$, $\omega = 1.9 = \omega_{1.9}$, $\omega = 3.2 \triangleq \omega_{3.2}$, and $\omega = 140 \triangleq \omega_{140}$ shown in (2) are used as the dominant data to determine x_l and y_l. Using the above design procedure, we can determine a set of equations $f_i(x_l, y_l) = 0$, $i = 1, 2, \cdots, 7$. These equations can be solved by using the Newton–Raphson method, with the set of initial estimates obtained from (9c). The data obtained from (12) at $\omega = 0.01 \triangleq \omega_{0.01}$, $\omega_{1.9}$, $\omega_{3.2}$, and ω_{140} are used in (9c) to determine the initial estimates x_l^* and y_l^*. Using these values as initial values for the Newton–Raphson method, gives the desired constants x_l and y_l at the 17th iteration with error tolerance of 10^{-6}. The newly designed digital controller $G_c^*(z)$ is

$$G_c^*(z) = \frac{13.170\,704z^3 - 25.531\,430z^2 + 14.629\,635z - 2.268\,545\,1}{z^3 - 0.374\,248\,41z^2 - 0.327\,570\,47z - 0.297\,947\,56}. \tag{16}$$

The closed-loop pulse-transfer function is

$$\frac{Y(z)}{R(z)} \triangleq T_{e2}(z) = \frac{G_{e2}(z)}{1 + G_{e2}(z)} \tag{17}$$

where

$$G_{e2}(z) = G_c^*(z)G_hG_0^*(z).$$

The Nyquist plot of $G_{e2}(z)$ shown in Fig. 2 matches very well that of $G_e(s)$. The unit-step response for $Y(z)$ in (17) is shown in Fig. 3. The time response of $T_{e2}(z)$ very closely matches that of the original system $T_e(s)$. The resulting design is seen to be quite satisfactory.

V. CONCLUSION

A dominant-data matching method has been given for fitting the coefficients of a pulse-transfer function from available time and frequency response data or from assigned design goals expressed by a set of control specifications. When the dominant data are obtained from a high-order continuous-data as well as a discrete-data system, our new method has been used to determine a reduced-order discrete-data system. If the data are experimental time and frequency response data of a system to be identified, our method may be used to identify the pulse-transfer function. Also, the method has been used for redesigns of a continuous system using a digital filter with various sampling periods. The pulse-transfer function obtained by our new method has the exact dominant performance of the original or desired system. We feel that the flexibility and accuracy of our new method will have significant practical advantages for the design of digital control systems and for the implementation of a low-order digital controller by using low cost microprocessors.

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